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SPECTRUM SENSING ALGORITHM BASED ON SHAPIRO WILK TEST

ABSTRACT

In this study, spectrum algorithm based on Shapiro Wilk test is discussed and a spectrum detection method is proposed for Cognitive Radios with Shapiro Wilk test. In this method, it is tested whether the energy of the received signal has a Gauss distribution. Simulation studies were carried out with different parameters to evaluate the performance of the proposed detection method. It has been compared with the Energy detector in simulation studies. Under some conditions, the proposed method was found to be more successful than ED.

Keywords: Cognitive Radio, Normality Test, Shapiro Wilk Test, Spectrum Sensing, Spectrum Efficiency

1. INTRODUCTION

Hypothesis tests are statistical methods that are frequently used in engineering, medicine and biomedical statistics [1, 2 and 3]. Hypothesis tests are used to diagnose diseases in medicine and to make inferences such as confidence tests in statistics [4 and 5]. They are used for signal detection in radar and communication systems. Hypothesis testing radar and Cognitive Radios (CR) are also used for spectrum detection [6, 7, 8 and 9]. Although Neyman-Pearson and Generalized Maximum Likelihood tests are common in these systems, Rao, Minimax and T tests are also used in some systems [10, 11 and 12]. It can be said that NP and GLRD are better than other tests in terms of performance and reliability. It has been used for spectrum detection in some methods based on deep learning [13].

In hypothesis testing, it is investigated which of the hypotheses and the opposite hypothesis are better compatible with the result obtained from the sample. Of the two hypotheses being compared, one is called the null hypothesis and the other is called the alternative hypothesis [14]. For example, in spectrum sensing and radar applications, the null hypothesis = no signal or only noise, while the opposite hypothesis means that there is noise with the signal [15]. The hypothesis test was first used in the Energy Detector (ED) in the field of spectrum detection [16]. Here, the energy of the signal received by Cognitive Users (CU) is used as a test statistic and compared with the threshold. Thus, spectrum sensing is done. In CR systems, the cognitive or secondary user is usually the person who aims to receive the spectrum and opportunistically use the empty spectrum regions it finds [17]. ED-based sensing has subsequently been combined with many different methods to improve performance. In addition, cooperative detection methods aiming to detect the same spectrum by more than one user collaboratively are also available in the literature [18]. Rao test, one of the hypothesis tests, was used for spectrum detection in CR systems [19]. However, this test cannot be said to be very convenient in terms of performance and transaction cost. Neyman-Pearson and the Generalized Likelihood Ratio

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Test are also widely used in spectrum detection. In addition, these tests can be considered quite successful in terms of performance [20]. In this study, we propose a spectrum detection method based on Shapiro Wilk hypothesis testing [21 and 22]. This test is actually a test of normality. However, the disadvantage of this test is that this test cannot be applied to very long observation vectors. In the study, a spectrum detection study based on the ED detector was carried out under the Rayleigh fading channel. The perception model is given and the necessary theoretical findings are made. Simulation studies were carried out with the help of MATLAB ® to test the success of the proposed detector.

2. RESEARCH SIGNIFICANCE

Hypothesis tests are decision tests used in many fields such as engineering, medicine, statistics. Generally, hypothesis tests are used in decision making with two options. In electronic and communication engineering, hypothesis tests are used in radar and signal detection. The Shapiro Wilk test, on the other hand, is a test of normality in general. It determines whether a probability distribution has a Gaussian.

The Shapiro Wilk test is one of the statistical analysis methods that tests for normality. Due to some disadvantages of this method, it is generally used in fields such as economics and business. In this study, this test is used for spectrum detection in the ED-based spectrum detection method. For this reason, besides being the first in this field, it is thought that it will be a source for future studies. The significance of the study can be expressed in the following sentences.

Highlights:

- Emphasizes the importance of spectrum efficiency
- It proposes a new spectrum detection method.
- It supports the theoretical findings with simulation studies.

3. DETECTION MODEL AND PROPOSED DETECTOR

Spectrum sensing in Cognitive Radio Systems (CRS) can be roughly divided into two parts: centralized and distributed. Centralized sensing must have one Primary Base Station (PBS) and at least two CR users. CR users query the active or passive status of the PBS through the signals they detect. If PBS is passive, they use the available spectrum until PBS is active. Centralized sensing is mostly used in CR systems due to the advantages it provides. For centralized and distributed sensing, see figure 1.

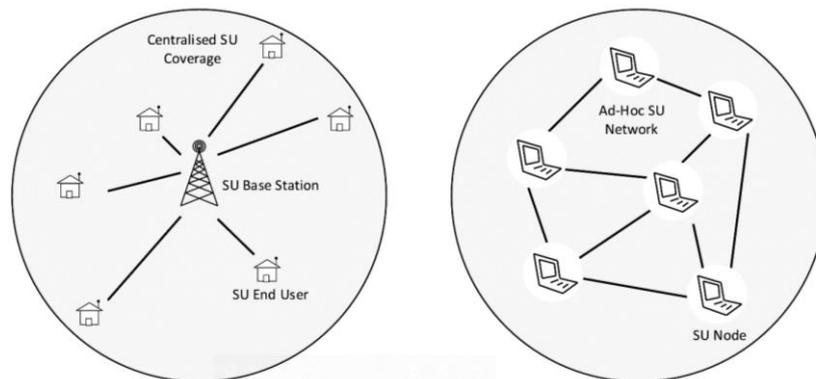


Figure 1. Central(right) and distributed (left) spectrum sensing

In centralized spectrum sensing, CR users are tasked with determining whether the primary transmitter is active/passive. So if the



PBS is passive they will use this region of the spectrum. When PBS is active, they will empty this area immediately. This function of CR users is defined as spectrum sensing and can be expressed as follows.

$$H_0 \rightarrow y(n) = w(n) \quad (1)$$

$$H_1 \rightarrow y(n) = x(n)H + w(n) \quad (2)$$

Suppose H_1 and H_0 in Equ. 1 and Equation 2 indicate that the PBS is active and passive. Let $x(n)$, $y(n)$ represent the PBS signal and the signal detected by the CR user, respectively. Then, according to Equation. 1, when PBS is passive, CR users will only detect noise. In addition $w(n)$ is the, which is having mean zero and variance, namely $w(n) \sim N(0, \sigma_w^2)$.

$$H_0: y(n) = \omega(n) \quad (3)$$

$$H_1: y(n) = h(n)x(n) + \omega(n) \quad (4)$$

Where $y(n), x(n), w(n)$ represent received signal samples by Cognitive User (CU), primary user signal and white Gaussian noise, respectively. Besides $h(n)$ is channel coefficient matrix (Rayleigh Channel). It should be noted that H_0 indicates that the primary transmitter is passive and H_1 indicates that it is active. Then ED method is expressed mathematically as follows.

$$T_i(y) = \sum_{n=1}^N |y(n)|^2 < \zeta \text{ under } H_0 \quad (5)$$

$$T_i(y) = \sum_{n=1}^N |y(n)|^2 > \zeta \text{ under } H_1 \quad (6)$$

Where $T_i(y)$ denotes Test Statistic (TS) for ED detector. ζ is the threshold is expressed as follows.

$$\zeta = \sigma_w^2 (Q^{-1}(P_{fa}) \sqrt{2N} + n) \quad (7)$$

Where σ_w^2 and Q^{-1} indicates noise variance and inverse complementary Q function, respectively.

In fact, the Shapiro Wilk test is a statistical hypothesis test used for normality testing. For spectrum sensing, the numerical values of T_i should be ordered from largest to smallest. Namely;

$$T_{i_1}(y) < T_{i_2}(y) < T_{i_3}(y) < \dots < T_{i_N}(y) \quad (8)$$

Then;

$$\theta^2 = \sum_{n=1}^N (T_i - \bar{T})^2 \quad (9)$$

Where \bar{T} indicates $1/N \sum_{i=1}^N (T_i)$. In the next step, the ζ parameter is calculated and the test statistics for the Shapiro Wilk test are obtained [21].

$$\zeta = \sum_{n=1}^K a_{N-i+1} (T_{N-i+1} - T_i)^2 \quad (10)$$

Where a_N are given in Table 1. Finally, the test statistic for the Shapiro Wilk test is calculated as follows.

$$TS_{SW} = \frac{\sum_{n=1}^N (T_i - \bar{T})^2}{\sum_{n=1}^K a_{N-i+1} (T_{N-i+1} - T_i)^2} \quad (11)$$



Table 1. Numerical parameter for Shapiro Wilk Test [23]

n	2	3	4	5	6	7	8	9	10	11	12	13	14
a1	0.7071	0.7071	0.6872	0.6646	0.6431	0.6233	0.6052	0.5888	0.5739	0.5601	0.5475	0.5359	0.5221
a2			0.1677	0.2413	0.2805	0.3031	0.3164	0.3244	0.3291	0.3315	0.3325	0.3325	0.3318
a3					0.0875	0.1401	0.1743	0.1976	0.2141	0.226	0.2347	0.2412	0.246
a4							0.0561	0.0947	0.1224	0.1429	0.1586	0.1707	0.1802
a5									0.0399	0.0595	0.0922	0.1099	0.124
a6											0.0803	0.0539	0.0727
a7													0.024

The final step for spectrum decision in the Shapiro Wilk test is with the following calculation.

$$\text{Spectrum Decision} = \begin{cases} H_0 & \text{if } TS > W_{\text{table}} \\ H_1 & \text{if } TS < W_{\text{table}} \end{cases} \quad (12)$$

Here it is given by source W_{table} [23]. In order to benefit from this table, it is necessary to know the sample length.

4. FINDINGS AND DISCUSSIONS

In this study, MATLAB-based simulation studies were carried out to determine the detection performance of the proposed method. The detection scenario was determined as centralized spectrum sensing (Figure 1).

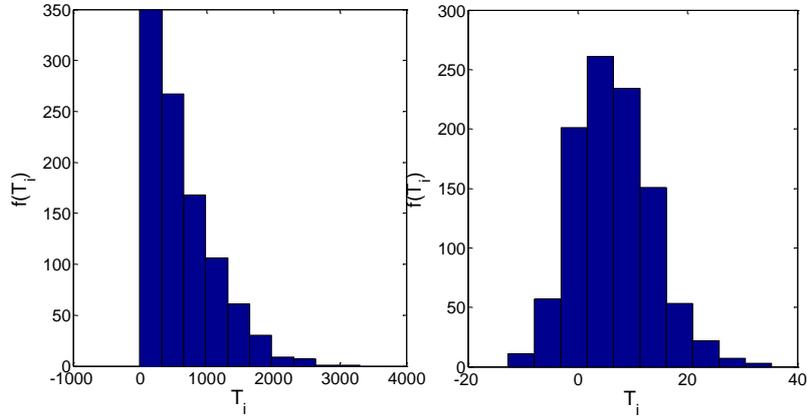


Figure 2. Rayleigh and Gaussian probability density functions

The PSB signal is randomly generated and transmitted through the Rayleigh fading channel to try to detect this signal at different signal noise ratios by the CR user. In addition, channel coefficient matrix and PBS signal are generated randomly. And for the Monte Carlo analysis, the detection methods were run 500 times and averaged. First of all, it should not be forgotten that the Shapiro Wilk test is a test of normality.



That is, according to the detection code, the T_i parameter will have different probability distributions according to different SNRs. These differences are given in Figure 2.

In this study, the results were compared with the ED-based sensing method. Since ED is the most widely used detection method, comparison is made with this method. In addition, the ED-based detection method is the spectrum detection method with the least processing cost and detection time. In Figure 2, the detection performances for the ED and the proposed method are given. The sample length for ED is 1000. For the recommended method, it is 100 and 1000. The choice of sample length is very important in spectrum sensing because as the sample length decreases, the detection time will also decrease. This is because the transaction cost is reduced. The ED method is computationally cost-effective but vulnerable to noise immunity. So it is very much affected by noise uncertainty. Knowing the noise variance while calculating the threshold value in the ED method leads to this result. Because the noise hallucination cannot be known in the application, it is therefore estimated. Estimation errors that will occur reduce the success of the ED method.

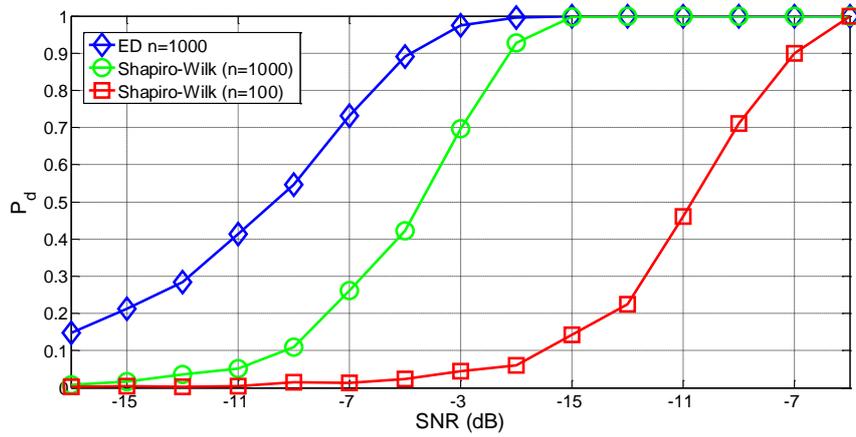


Figure 3. SNR versus P_d for ED and Shapiro Wilk test

However, since the proposed method is a normality test, it does not need to know the noise variance. This phenomenon can be cited as an advantage of the Shapiro Wilk test. For the same sample length in Figure 3, the ED method is more successful than the proposed method. However, the noise uncertainty factor is not taken into account in Figure 3. Another conclusion to be drawn from Figure 3 is that, as in all other spectrum detection methods, the proposed method is directly proportional to the detection performance of the sample length. This phenomenon is not seen as a negative for the proposed method because it is the case with all spectrum detection methods.

Figure 4 shows the detection performances at different sample lengths for the proposed Shapiro Wilk test. As can be seen from the graph, the decrease in sample length affected the detection performance negatively. For the proposed detector, when the sample length is 1000, the spectrum is perceived correctly in the presence of approximately -13 dB of noise. This point is circled in red in Figure 4.

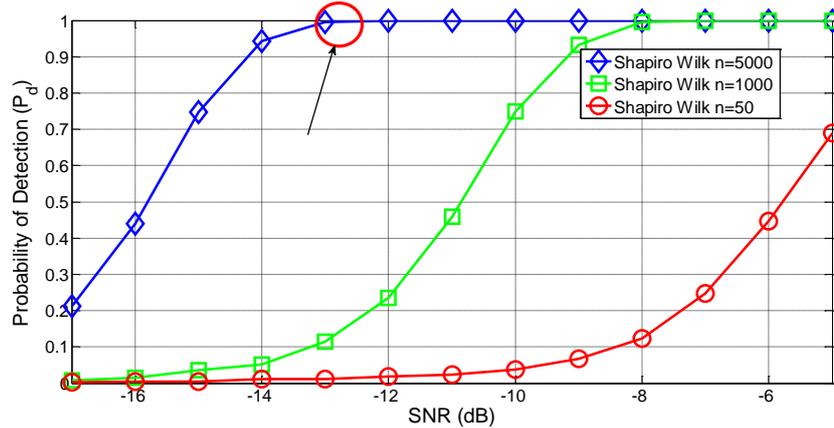


Figure 4. SNR versus Pd for Shapiro Wilk test

Table 2 gives the detection times of the ED with the proposed detector for different sample lengths. Detection time is very important in CR systems and it is desired to be very short. A shorter detection time will increase the spectrum efficiency. Also, if this period is long, it will cause interference with the CR user when the primary user enters the spectrum.

Table 2. Sensing times for ED and Shapiro Wilk Test

	n=100	n=1000
ED	3.5 ms	12.0 ms
Shapiro Wilk test	2.1 ms	8.3 ms

Compared to the proposed detector ED, it seems to be more advantageous in terms of detection time. Because the operation cost of the proposed detector is very low. Although MATLAB is used for simulation studies in this study, there are some online websites for this test [24].

5. CONCLUSION AND RECOMMENDATIONS

In this study, a Shapiro Wilk based spectrum detection method, which is a test of normality, is proposed. In the study, spectrum detection was performed under Rayleigh fading channels by using the central spectrum detection method. Necessary theoretical analyzes of the proposed detector were made and simulation studies were carried out to prove these studies. According to the simulation results, the proposed detector is more successful than ED, especially in terms of detection time.

CONFLICT OF INTEREST

The author declared no conflict of interest.

FINANCIAL DISCLOSURE

The author declare that this study has received no financial support.

DECLARATION OF ETHICAL STANDARDS

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.



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