ISSN:1306-3111
e-Journal of New World Sciences Academy
2010, Volume: 5, Number: 1, Article Number: 3A0018

PHYSICAL SCIENCES
Received: May 2009
Accepted: January 2010
Series : 3A
ISSN : 1308-7304

İbrahim Enam İnan
Yavuz Uğurlu
Bülent Kılıç
Firat University
ieinan@yahoo.com
Elazig-Turkey

$(G'/G)-$ EXPANSION METHOD FOR THE TRAVELLING WAVE SOLUTIONS OF FIFTH ORDER KDV EQUATION AND KAUP-KUPERSHMIDT EQUATION

ABSTRACT

In this paper, we implemented the (G'/G)- expansion method the traveling wave solutions of the fifth order KdV equation and Kaup-Kupershmidt equation. By using this scheme, we found some traveling wave solutions of the above-mentioned equations.

Keywords: Fifth Order KdV Equation, Kaup-Kupershmidt Equation, (G'/G)-Expansion Method, Traveling Wave Solutions, Nonlinear Partial Differential Equations

BEŞİNCİ MERTEBEDEN KdV DENKLEMİ VE KAUP-KUPERSHMIDT DENKLEMİNİN HAREKET EDEN DALGA ÇÖZÜMLERİ İÇİN (G'/G)-AÇILIM METODU

ÖZET

Bu çalışmada beşinci mertebeden KdV denklemi ve Kaup-Kupershmidt denkleminin hareket eden dalga çözümleri için (G'/G) - açılım metodu sunacağız. Bu tekniği kullanarak beşinci mertebeden KdV denklemi ve Kaup-Kupershmidt denkleminin birkaç tane hareket eden dalga çözümlerini bulacağız.

Anahtar Kelimeler: Beşinci Mertebeden KdV Denklemi, Kaup-Kupershmidt Denklemi, (G'/G)- Açılım Metodu, Hareket Eden Dalga Çözümleri, Lineer Olmayan Kısmi Diferensiyel Denklemler

1. INTRODUCTION (GİRİŞ)

Nonlinear phenomena play a crucial role in applied mathematics and physics. Calculating exact and numerical solutions, in particular, traveling wave solutions, of nonlinear equations in mathematical physics plaices an important role in soliton theory [1 and 2]. Recently, it has become more interesting that obtaining exact solutions of nonlinear partial differential equations through using symbolical computer programs such as Maple, Matlab, Mathematica that facilitate complex and tedious algebraical computations. It is too important to find exact solutions of nonlinear partial differential equations. These equations are mathematical models of complex physical occurrences that arise in engineering, chemistry, biology, mechanics and physics. Various effective methods have been developed to understand the mechanisms of these physical models, to help physicans and engineers and to ensure knowledge for physical problems and its applications. Many explicit exact methods have been introduced in literature [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 and 23]. Some of them are: Bäcklund transformation, generalized Miura transformation, Darboux transformation, Cole-Hopf transformation, tanh method, sine-cosine method, Painlevé method, homogeneous balance method, similarity reduction method and so on.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this work, we will consider to solve the traveling wave solutions of the fifth order KdV equation and Kaup-Kupershmidt equation by using the (G'/G)- expansion method which is introduced by Mingliang Wang, Xiangzheng Li and Jinliang Zhang [17].

3. METHOD AND ITS APPLICATIONS (YÖNTEM VE UYGULAMALARI)

Before starting to give the (G'/G)- expansion method, we will give a simple description of the (G'/G)- expansion method. For doing this, one can consider in a two-variables general form of nonlinear PDE

$$Q(u,u_t,u_x,u_{xx},\ldots)=0\ , \tag{1}$$

and transform Eq.(1) with $u(x,t)=u(\xi),\ \xi=x-wt$, where w is constant. After transformation, we get a nonlinear ODE for $u(\xi)$

$$Q'(u',u'',u''',\ldots)=0. \tag{2}$$

The solution of the equation (2) we are looking for is expressed as

$$u(\xi)=\alpha_m\left(\frac{G'}{G}\right)^m+\cdots, \tag{3}$$

where $G=G(\xi)$ satisfies the second order LODE in the form

$$G''+\lambda G'+\mu G=0, \tag{4}$$

where $\alpha_m,\ldots,\lambda$ and μ are constants to be determined later, $\alpha_m\neq 0,$ the positive integer m can be determined by balancing the highest order derivative and with the highest nonlinear terms into equation (2). Substituting solution (3) into equation (2) and using (4) yields a set of algebraic equations for same order of (G'/G); then all coefficients same order of (G'/G) have to vanish. After this separated algebraic equation, we can find $\alpha_m,\ldots,w,\lambda$ and μ constants. General solutions of the equation (4) have been well known us, then substituting $\alpha_m,\ldots,w$ and the general solutions of equation (4) into (3) we have more traveling wave solutions of equation (1) [17].

3. EXAMPLE 1. (ÖRNEK 1)

Consider the fifth order KdV equation,

$$u_t + u_x + uu_x + u_{xxx} + uu_{xxx} + u_{xxxxx} = 0. \qquad (5)$$

For doing this example, we can use transformation with Eq. (1) then Eq. (5) become

$$-wu' + u' + uu' + u''' + uu''' + u^{(5)} = 0, \qquad (6)$$

When balancing uu''' with $u^{(5)}$ then gives $m=2$. Therefore, we may choose

$$u(\xi) = \alpha_2\left(\frac{G'}{G}\right)^2 + \alpha_1\left(\frac{G'}{G}\right) + \alpha_0, \qquad (7)$$

Substituting equation (7) into (6) yields a set of algebraic equations for $\alpha_0, \alpha_1, \alpha_2$ and w. These systems are

$$\begin{aligned}
&-\alpha_1\mu + w\alpha_1\mu - \alpha_0\alpha_1\mu - \alpha_1\lambda^2\mu - \alpha_0\alpha_1\lambda^2\mu - \alpha_1\lambda^4\mu - 2\alpha_1\mu^2 - 2\alpha_0\alpha_1\mu^2 - 6\alpha_2\lambda\mu^2\\
&-6\alpha_0\alpha_2\lambda\mu^2 - 22\alpha_1\lambda^2\mu^2 - 30\alpha_2\lambda^3\mu^2 - 16\alpha_1\mu^3 - 120\alpha_2\lambda\mu^3 = 0,\\
&-\alpha_1\lambda + \alpha_1\lambda w - \alpha_0\alpha_1\lambda - \alpha_1\lambda^3 - \alpha_0\alpha_1\lambda^3 - \alpha_1\lambda^5 - \alpha_1^2\mu - 2\alpha_2\mu + 2\alpha_2\mu w - 2\alpha_0\alpha_2\mu\\
&-8\alpha_1\lambda\mu - 8\alpha_0\alpha_1\lambda\mu - \alpha_1^2\lambda^2\mu - 14\alpha_2\lambda^2\mu - 14\alpha_0\alpha_2\lambda^2\mu - 52\alpha_1\lambda^3\mu - 62\alpha_2\lambda^4\mu\\
&-2\alpha_1^2\mu^2 - 16\alpha_2\mu^2 - 16\alpha_0\alpha_2\mu^2 - 136\alpha_1\lambda\mu^2 - 6\alpha_1\alpha_2\lambda\mu^2 - 584\alpha_2\lambda^2\mu^2 - 272\alpha_2\mu^3 = 0,\\
&-\alpha_1 + \alpha_1 w - \alpha_0\alpha_1 - \alpha_1^2\lambda - 2\alpha_2\lambda + 2\alpha_2\lambda w - 2\alpha_0\alpha_2\lambda - 7\alpha_1\lambda^2 - 7\alpha_0\alpha_1\lambda^2 - \alpha_1^2\lambda^3 - 8\alpha_2\lambda^3\\
&-8\alpha_0\alpha_2\lambda^3 - 31\alpha_1\lambda^4 - 32\alpha_2\lambda^5 - 8\alpha_1\mu - 8\alpha_0\alpha_1\mu - 3\alpha_1\alpha_2\mu - 8\alpha_1^2\lambda\mu - 52\alpha_2\lambda\mu\\
&-52\alpha_0\alpha_2\lambda\mu - 292\alpha_1\lambda^2\mu - 15\alpha_1\alpha_2\lambda^2\mu - 884\alpha_2\lambda^3\mu - 136\alpha_1\mu^2 - 18\alpha_1\alpha_2\mu^2 - 1712\alpha_2\lambda\mu^2\\
&-6\alpha_2^2\lambda\mu^2 = 0, -60\alpha_1\alpha_2\lambda\mu - 3104\alpha_2\lambda^2\mu - 14\alpha_2^2\lambda^2\mu - 1232\alpha_2\mu^2 - 16\alpha_2^2\mu^2 = 0,\\
&-6\alpha_1 - 6\alpha_0\alpha_1 - 3\alpha_1\alpha_2 - 12\alpha_1^2\lambda - 54\alpha_2\lambda - 54\alpha_0\alpha_2\lambda - 2\alpha_2^2\lambda - 390\alpha_1\lambda^2 - 45\alpha_1\alpha_2\lambda^2\\
&-1710\alpha_2\lambda^3 - 8\alpha_2^2\lambda^3 - 240\alpha_1\mu - 48\alpha_1\alpha_2\mu - 3960\alpha_2\lambda\mu - 52\alpha_2^2\lambda\mu = 0,\\
&-\alpha_1^2 - 2\alpha_2 + 2\alpha_2 w - 2\alpha_0\alpha_2 - 12\alpha_1\lambda - 12\alpha_0\alpha_1\lambda - 3\alpha_1\alpha_2\lambda - 7\alpha_1^2\lambda^2 - 38\alpha_2\lambda^2 - 38\alpha_0\alpha_2\lambda^2\\
&-180\alpha_1\lambda^3 - 9\alpha_1\alpha_2\lambda^3 - 422\alpha_2\lambda^4 - 8\alpha_1^2\mu - 40\alpha_2\mu - 40\alpha_0\alpha_2\mu - 2\alpha_2^2\mu - 480\alpha_1\lambda\mu\\
&-6\alpha_1^2 - 24\alpha_2 - 24\alpha_0\alpha_2 - 2\alpha_2^2 - 360\alpha_1\lambda - 66\alpha_1\alpha_2\lambda - 3000\alpha_2\lambda^2 - 38\alpha_2^2\lambda^2 - 1680\alpha_2\mu\\
&-40\alpha_2^2\mu = 0, -120\alpha_1 - 30\alpha_1\alpha_2 - 2400\alpha_2\lambda - 54\alpha_2^2\lambda = 0, -720\alpha_2 - 24\alpha_2^2 = 0.
\end{aligned} \qquad (8)$$

From the solutions system, we obtain the following with the aid of Mathematica.

$$\alpha_0 = \frac{1}{2}\left(3 - 5\lambda^2 - 40\mu\right), \quad \alpha_1 = -30\lambda, \quad \alpha_2 = -30, \quad w = \frac{5 - 3\lambda^4 + 24\lambda^2\mu - 48\mu^2}{2} \qquad (9)$$

Substituting (9) into (7) we have three types of traveling wave solutions of equation (5):

(i) When $\lambda^2 - 4\mu > 0$,

$$u_1(\xi) = \frac{3}{2} - 30\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\frac{C_1 Sinh[\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi] + C_2 Cosh[\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi]}{C_1 Cosh[\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi] + C_2 Sinh[\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi]}\right)\right)^2 + 5\lambda^2 - 20\mu.$$

where $\xi = \left[x - \left(\frac{5 - 3\lambda^4 + 24\lambda^2\mu - 48\mu^2}{2}\right)t\right]$, C_1 and C_2 are arbitrary constants.

(ii) When $\lambda^2 - 4\mu < 0$,

$$u_2(\xi)=\frac{3}{2}-30\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1 Sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+C_2 Cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{C_1 Cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]+C_2 Sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}\right)\right)^2+5\lambda^2-20\mu.$$

where $\xi=\left[x-\left(\frac{5-3\lambda^4+24\lambda^2\mu-48\mu^2}{2}\right)t\right]$, C_1 and C_2 are arbitrary constants.

(iii) When $\lambda^2-4\mu=0$,

$$u_3(\xi)=\frac{3}{2}-30\left(\frac{C_2}{C_1+C_2\left(x-\frac{5}{2}t\right)}\right)^2$$

where C_1 and C_2 are arbitrary constants.

EXAMPLE 2. (ÖRNEK 2)

Consider Kaup-Kupershmidt equation,

$$u_t+45u_x u^2-\frac{75}{2}u_x u_{xx}-15uu_{xxx}+u_{xxxxx}=0. \tag{10}$$

For doing this example, we can use transformation with Eq. (1) then Eq. (10) become

$$-wu'+45u'u^2-\frac{75}{2}u'u''-15uu'''+u^{(5)}=0. \tag{11}$$

When balancing $u'u^2, u'u''$ and uu''' with $u^{(5)}$ then gives *n=2*. Therefore, we may choose

$$u(\xi)=\alpha_2\left(\frac{G'}{G}\right)^2+\alpha_1\left(\frac{G'}{G}\right)+\alpha_0, \tag{12}$$

Substituting equation (12) into (11) yields a set of algebraic equations for $\alpha_0,\alpha_1,\alpha_2$ and w. These systems are

$$\alpha_1\mu w-45\alpha_0^2\alpha_1\mu+15\alpha_0\alpha_1\lambda^2\mu-\alpha_1\lambda^4\mu+30\alpha_0\alpha_1\mu^2+\frac{75}{2}\alpha_1^2\lambda\mu^2+90\alpha_0\alpha_2\lambda\mu^2$$
$$-22\alpha_1\lambda^2\mu^2-30\alpha_2\lambda^3\mu^2-16\alpha_1\lambda^3+75\alpha_1\alpha_2\mu^3-120\alpha_2\lambda\mu^3=0,$$
$$\alpha_1\lambda w-45\alpha_0^2\alpha_1\lambda+15\alpha_0\alpha_1\lambda^3-\alpha_1\lambda^5-90\alpha_0\alpha_1^2\mu+2\alpha_2\mu w-90\alpha_0^2\alpha_2\mu+120\alpha_0\alpha_1\mu\lambda$$
$$+90\alpha_1^2\mu\lambda^2+210\alpha_0\alpha_2\mu\lambda^2-52\alpha_1\lambda^3\mu-62\alpha_2\mu\lambda^4+105\alpha_1^2\mu^2+240\alpha_0\alpha_2\mu^2-136\alpha_1\mu^2\lambda$$
$$+465\alpha_1\alpha_2\mu^2\lambda-584\alpha_2\mu^2\lambda^2-272\alpha_2\mu^3+150\alpha_2^2\mu^3=0,$$
$$\alpha_1 w-45\alpha_0^2\alpha_1-90\alpha_0\alpha_1^2+2\alpha_2\lambda w-90\alpha_0^2\alpha_2\lambda+105\alpha_0\alpha_1\lambda^2+\frac{105}{2}\alpha_1^2\lambda^3+120\alpha_0\alpha_2\lambda^3-31\alpha_1\lambda^4$$
$$-32\alpha_2\lambda^5+120\alpha_0\alpha_1\mu-45\alpha_1^3\mu-270\alpha_0\alpha_1\alpha_2\mu+345\alpha_1^2\lambda\mu+780\alpha_0\alpha_2\lambda\mu-292\alpha_1\lambda^2\mu$$
$$+750\alpha_1\alpha_2\lambda^2\mu-884\alpha_2\lambda^3\mu-136\alpha_1\lambda^2+795\alpha_1\alpha_2\mu^2-1712\alpha_2\lambda\mu^2+690\alpha_2^2\lambda\mu^2=0,$$
$$-90\alpha_0\alpha_1^2+2\alpha_2 w-90\alpha_0^2\alpha_2+180\alpha_0\alpha_1\lambda-45\alpha_1^3\lambda-270\alpha_0\alpha_1\alpha_2\lambda+255\alpha_1^2\lambda^2+570\alpha_0\alpha_2\lambda^2$$
$$-180\alpha_1\lambda^3+360\alpha_1\alpha_2\lambda^3-422\alpha_2\lambda^4+270\alpha_1^2\mu+600\alpha_0\alpha_2\mu-180\alpha_1^2\alpha_2\mu-180\alpha_0\alpha_2^2\mu$$
$$-480\alpha_1\lambda\mu+2250\alpha_1\alpha_2\lambda\mu-3104\alpha_2\lambda^2\mu+960\alpha_2^2\lambda^2\mu-1232\alpha_2\mu^2+990\alpha_2^2\mu^2=0,$$

$$90\alpha_0\alpha_1 - 45\alpha_1^3 - 270\alpha_0\alpha_1\alpha_2 + \frac{735}{2}\alpha_1^2\lambda + 810\alpha_0\alpha_2\lambda - 180\alpha_1^2\alpha_2\lambda - 180\alpha_0\alpha_2^2 - 390\alpha_1\lambda^2$$
$$+1500\alpha_1\alpha_2\lambda^2 - 1710\alpha_2\lambda^3 + 420\alpha_2^2\lambda^3 - 240\alpha_1\mu + 1545\alpha_1\alpha_2\mu - 225\alpha_1\alpha_2^2\mu - 3960\alpha_2\lambda\mu$$
$$+2580\alpha_2^2\lambda\mu = 0,$$
$$165\alpha_1^2 + 360\alpha_0\alpha_2 - 180\alpha_1^2\alpha_2 - 180\alpha_0\alpha_2^2 - 360\alpha_1\lambda + 1965\alpha_1\alpha_2\lambda - 225\alpha_1\alpha_2^2\lambda - 3000\alpha_2\lambda^2$$
$$+1620\alpha_2^2\lambda^2 - 1680\alpha_2\mu + 1650\alpha_2^2\mu - 90\alpha_2^3\mu = 0,$$
$$-120\alpha_1 + 825\alpha_1\alpha_2 - 225\alpha_1\alpha_2^2 - 2400\alpha_2\lambda + 2010\alpha_2^2\lambda - 90\alpha_2^3\mu = 0,\ -720\alpha_2 + 810\alpha_2^2 - 90\alpha_2^3 = 0. \quad (13)$$

From the solutions system, we obtain the following with the aid of Mathematica.

$$\alpha_0 = \frac{1}{12}\left(\lambda^2 + 8\mu\right), \quad \alpha_1 = \lambda, \quad \alpha_2 = 1, \quad w = \frac{\lambda^4 - 8\lambda^2\mu + 16\mu^2}{16} \quad (14)$$

Substituting (14) into (12) we have three types of traveling wave solutions of equation (10):

(i) When $\lambda^2 - 4\mu > 0$,

$$u_1(\xi) = -\frac{\lambda^2}{6} + \frac{2\mu}{3} + \left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A_1 Sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + A_2 Cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}{A_1 Cosh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi] + A_2 Sinh[\frac{\sqrt{\lambda^2-4\mu}}{2}\xi]}\right)\right)^2$$

where $\xi = \left[x - \left(\frac{\lambda^4 - 8\lambda^2\mu + 16\mu^2}{16}\right)t\right]$, A_1 and A_2 are arbitrary constants.

(ii) When $\lambda^2 - 4\mu < 0$,

$$u_2(\xi) = -\frac{\lambda^2}{6} + \frac{2\mu}{3} + \left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A_1 Sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + A_2 Cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}{A_1 Cos[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi] + A_2 Sin[\frac{\sqrt{4\mu-\lambda^2}}{2}\xi]}\right)\right)^2$$

where $\xi = \left[x - \left(\frac{\lambda^4 - 8\lambda^2\mu + 16\mu^2}{16}\right)t\right]$, A_1 and A_2 are arbitrary constants.

(iii) When $\lambda^2 - 4\mu = 0$,

$$u_3(\xi) = \left(\frac{A_2}{A_1 + A_2 x}\right)^2$$

where A_1 and A_2 are arbitrary constants.

4. CONCLUSION (SONUÇ)

In this work, we will consider to solve the traveling wave solutions of the fifth order KdV equation and Kaup-Kupershmidt equation by using the (G'/G)- expansion method. The method [17] can be used to many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

REFERENCES (KAYNAKLAR)

1. Debtnath, L., (1997). Nonlinear Partial Differential Equations for Scientist and Engineers, Birkhauser, Boston, MA.
2. Wazwaz, A.M., (2002). Partial Differential Equations: Methods and Applications, Balkema, Rotterdam.
3. Hereman, W., Banerjee, P.P., Korpel, A., Assanto, G., van Immerzeele, A. and Meerpoel, A., (1986). J.Phys. A: Math. Gen., 19, 607.
4. Khater, A.H., Helal, M.A., and El-Kalaawy, O.H., (1998). Math. Methods Appl. Sci. 21, 719.
5. Wazwaz, A.M., (2001). Math. Comput. Simulation 56, 269.
6. Elwakil, S.A. , El-Labany, S.K. , Zahran, M.A., and Sabry, R., (2002). Phys. Lett. A. 299, 179.
7. Lei, Y., Fajiang, Z. and Yinghai, W., (2002). Chaos, Solitons & Fractals 13, 337.
8. Zhang, J.F., (1999). Int. J. Theor. Phys. 38, 1829.
9. Wang, M.L., (1996). Phys. Lett. A. 213, 279.
10. Wang, M.L., Zhou, Y.B. and Li, Z.B., (1996). Phys. Lett. A. 216, 67.
11. Malfliet, M.L., (1992). Am. J. Phys. 60, 650.
12. Parkes, E.J. and Duffy, B.R., (1996). Comput. Phys. Commun. 98, 288.
13. Duffy, B.R. and Parkes, E.J., (1996). Phys. Lett. A. 214,171.
14. Parkes, E.J. and Duffy, B.R., (1997). Phys. Lett. A. 229, 217.
15. Hereman, W.,(1991). Comput. Phys. Commun. 65, 143.
16. Fan, E.G., (2000). Phys. Lett. A. 277, 212.
17. Wang, M., Li, X., Zhang, J., (2008). Phys. Lett. A 372, 417.
18. Aslan, I., Öziş, T., (2009). Appl. Math. Comput. 209, 425.
19. Aslan, I., Öziş, T., (2009). Appl. Math. Comput. 211, 531.
20. Bekir, A., (2008), Phys. Lett. A. 372, 3400.
21. Zhang, S., Dong L., Ba, J-M., and Sun, Y-N.., (2009). Phys. Lett. A 373, 905.
22. Borhanifar, A. and Kabir, M.M., (2009). Journal of Comput. and Appl. Math. 229, 158.
23. Zhang, H., (2009). Commun. Nonlinear Sci Numer Simulat., 14, 3220.