ISSN:1306-3111
e-Journal of New World Sciences Academy
2010, Volume: 5, Number: 2, Article Number: 1A0071

ENGINEERING SCIENCES
Received: March 2009
Accepted: March 2010
Series : 1A
ISSN : 1308-7231

Aydın Saatcı
Serap Kahraman
Dokuz Eylul University
aydin.saatci@deu.edu.tr
serap.kahraman@deu.edu.tr
Izmir-Turkey

DYNAMIC RESPONSES OF FOUNDATION BEAMS SUBJECTED TO TRANSVERSE LOADING ON ELASTIC SOIL BY DIRAC DISTRIBUTION THEORY

ABSTRACT

In this study, the dynamic analysis of a free beam subjected to transverse forces and moments on an elastic soil are investigated. The foundation model is based on the Winkler hypothesis. Using Dirac distribution theory, concentrated disturbances on beams are transformed to distributed loads in order to be able to use the governing differential equation established for distributed loads. An illustrative example is presented in order to demonstrate the use of the study and some of the obtained results are given in tables and figures.

Keywords: Free Beam, Foundation On Elastic Soil, Concentrated Loads, Dirac Distribution Theory, Dynamic Response

TEKİL KUVVET VE MOMENT ETKİSİNDEKİ ELASTİK ZEMİNE OTURAN TEMEL KİRİŞLERİNİN DIRAC DAĞILIM TEORİSİ KULLANILARAK DİNAMİK ANALİZİ

ÖZET

Bu çalışmada, tekil kuvvet ve momentlere maruz elastik zemine oturan serbest bir kirişin dinamik analizi incelenmiştir. Temel modeli Winkler hipotezine dayalıdır. Yayılı yükler için belirlenen diferansiyel denklemin kullanılabilmesi amacıyla, kirişteki tekil zorlamalar Dirac dağılım teorisi kullanarak yayılı yüklere dönüştürülmüştür. Çalışmayı açıklamak amacıyla tanımlayıcı bir örnek sunulmuş ve elde edilen bazı sonuçlar tablo ve şekillerde verilmiştir.

Anahtar Kelimeler: Serbest Kiriş, Elastik Zemine Oturan Temel, Tekil Yükler, Dirac Dağılım Teorisi, Dinamik Tepki

1. INTRODUCTION (GİRİŞ)

There are various methods used in the analysis of continuous foundations as a beam resting on elastic soils. The most important two of them are the subgrade modulus method pertaining to the theory of the first order and the method of modulus of elasticity based on a second order theory. The former presents a model in which the soil is assumed as dense liquid while the latter offers an elastic solid model.

In the subgrade modulus method, proposed by Winkler [1], it is assumed that the deflection at any point of the beam on elastic soil is proportional to the pressure applied at that point and is independent of pressure acting at nearby points of the beam. In other words, in this method the beam is considered as if it is resting on infinitely long independent elastic springs with subgrade modulus [2]. In the elastic solid model, the effects of the neighboring points to the point in question are taken into account by Boussinesque's load-deformation relation in an isotropic elastic semi-space. In this case, the soil is characterized by its elastic properties, namely, elastic modulus and Poisson's ratio. However, the solution of the differential equation established for this model may present certain computational difficulties and approximate methods may be needed to involve for the solution.

However, both models do not represent the real soil exactly. It behaves neither as a dense liquid nor as an elastic solid. With a more realistic hypothesis, some researchers developed two-parameter models for the elastic soil [3, 4, 5, 6 and 7]. In comparison with the single parameter model, i.e. Winkler model, these two-parameter foundation models represent the foundation characteristics more accurately. Vallabhan and Daloglu had developed relations in which subgrade modulus varies with depth which is equivalent to the two-parameter Vlasov-Leontiev solution and can be used in classical Winkler model [8]. Misir improved the Equivalent Winkler Subgrade Modulus Method as an approach to the two-parameter model (Vlasov-Leontiev) to include definition of the subgrade modulus considering the influence of both multilayered soil profiles and increase in effective stresses [9].

In this paper, the subgrade modulus method is used, which is also preferred in practice for static problems due to its simplicity of mathematical formulation. One of the most important drawbacks of this method is difficulties in determining the modulus of subgrade reaction of the soil. The variation of contact pressure over the bearing area requires the variation of subgrade modulus as well; subgrade modulus depends not only on the physical characteristics of the soil but also on the foundation dimensions, the rigidity of the foundation, the distribution of loading on the superstructure and the thickness of the compressible layer which causes settlement. Therefore accurate determination of deflection of the foundation and stresses on the superstructure can only be possible by using these factors [10].

In addition to all of these, it is known that the subgrade modulus values for dynamic loading are different from those for static loading. Based on these facts, values for subgrade modulus should be determined by field tests conducted for different types of soils, different loading conditions and different loading areas. However in practice, except for very important structures, subgrade modulus values are taken from tables prepared for different soil types. In the subgrade modulus method, the rigidity of the superstructure, the stress distribution under the foundation base and lateral movement of the base soil are left out from the mathematical model.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In the presented study, the dynamic responses of a free beam subjected to transverse forces and moments on elastic soil are investigated. The governing differential equation is established for distributed loads, therefore, initially using Dirac distribution theory, the concentrated forces and moments on beams have been transformed to distributed loads.

3. MATHEMATICAL FORMULATION (MATEMATİKSEL FORMÜLASYON)

Fig. 1a shows a foundation beam with flexural rigidity *EI(x)*, coefficient of viscous damping *c(x)* per unit length, base width *b(x)*, cross-sectional area *A(x)*, mass density *ρ* and mass *m(x) = ρA(x)* per unit length on a soil with subgrade modulus K_0. The beam is subjected to distributed external load *f(x,t)* which may vary with position *x* and time *t*. The forces on a differential element of length *dx* are shown in Fig. 1b, where *V(x,t)* is the transverse shear force, *M(x,t)* is the bending moment, *y(x,t)* is the transverse displacement, $c(x)\partial y/\partial t$ is the viscous damping force, $m(x)\partial^2 y/\partial t^2$ is the inertia force and $k(x)y = K_0 b(x)y$ is the elastic response of the soil.

In the analysis the effects of shear and axial deformations and rotational inertia are ignored. The governing differential equation for the transverse vibration of a beam on elastic soil shown in Fig. 1 can be written as

$$\frac{\partial^2}{\partial x^2}(EI(x)\frac{\partial^2 y}{\partial x^2}) + m(x)\frac{\partial^2 y}{\partial t^2} + c(x)\frac{\partial y}{\partial t} + k(x)y = f(x,t) \tag{1}$$

The solution of this partial differential equation under the boundary and initial conditions yields the response $y(x,t)$ of the beam in position *x* and at time *t*. Once the deflection is determined, the slope, bending moment and the shear can be calculated by taking the first, second and third derivative of the solution (response) function with respect to *x*, respectively.

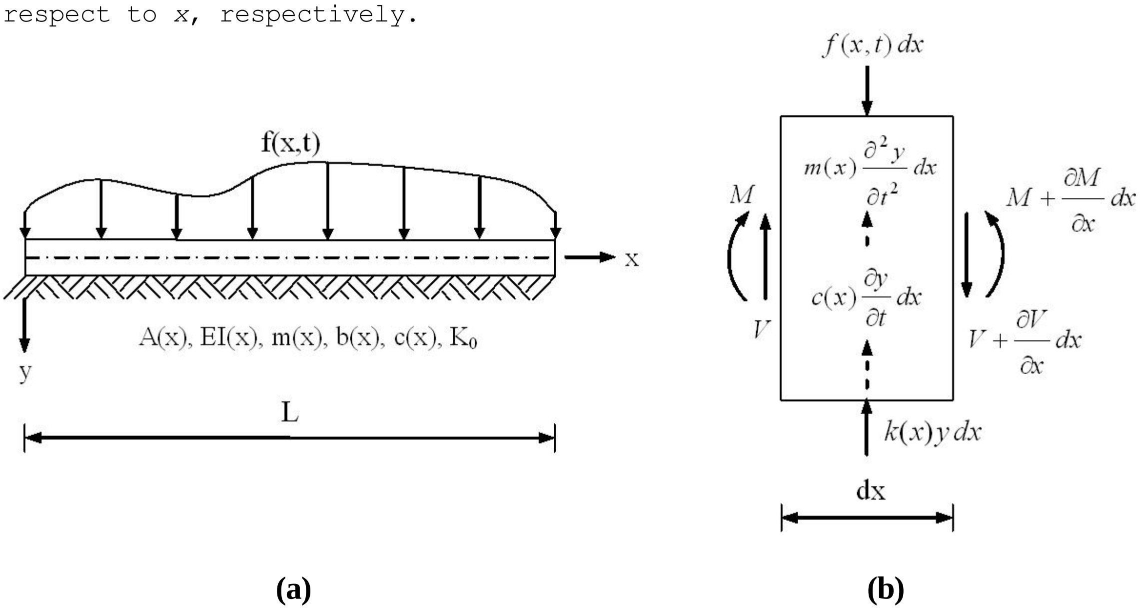

Figure 1. Foundation beam (a) subjected to distributed external dynamic loads, (b) the forces on a differential element
(Şekil 1. (a) Yayılı dış dinamik yükler etkisindeki temel kirişi, (b)diferansiyel eleman üzerinde kesit tesirleri)

3.1. Natural Frequencies and Mode Shapes (Doğal Açısal Frekanslar ve Mod Şekilleri)

For free vibration, $f(x,t)=0$, and with the assumption that the damping coefficient and the section characteristics are constant along the beam, the natural frequencies and modes can be obtained by the solution of the following homogeneous partial differential equation with constant coefficients

$$EI\frac{\partial^4 y}{\partial x^4}+m\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}+ky=0 \quad (2)$$

The solution function of Eq. (2) can be written by the method of separation of variables as

$$y(x,t)=X(x)\cdot T(t) \quad (3)$$

where $X(x)$ is the characteristic shape function and $T(t)$ is a time function. The substitution of Eq. (3) into Eq. (2) leads to

$$\frac{EIX^{IV}}{X}=-\frac{m\ddot{T}+c\dot{T}+kT}{T} \quad (4)$$

where Roman indices denote derivatives with respect to *x* and overdots indicate derivatives with respect to time. Since the left hand side of Eq. (4) is a function only of *x* while the right hand side is a function of *t* only, Eq. (4) is true only if each side is equal to the same constant. Designating this constant by *p* and setting both sides equal to it yields

$$EIX^{IV}-pX=0 \quad (5)$$

and

$$m\ddot{T}+c\dot{T}+(k+p)T=0 \quad (6)$$

The solution of Eq. (5) is

$$X(x)=C_1\sin\lambda x+C_2\cos\lambda x+C_3\sinh\lambda x+C_4\cosh\lambda x \quad (7)$$

where

$$\lambda=\sqrt[4]{\frac{p}{EI}} \quad (8)$$

The four integration constants in Eq. (7) are determined via the boundary conditions.

In the case of underdamped motion, the solution of Eq. (6) is

$$T(t)=e^{-\zeta\omega t}\left[T_0\cos(\omega_D t)+\frac{\dot{T}_0+\zeta\omega T_0}{\omega_D}\sin(\omega_D t)\right] \quad (9)$$

where T_0 and $\dot{T}_0$ are parameters which depend on initial conditions and ω_D is the damped natural frequency of the system which is given by

$$\omega_D=\omega\sqrt{1-\zeta^2} \quad (10)$$

where ω is the undamped natural frequency, namely

$$\omega=\sqrt{\frac{k+p}{m}} \quad (11)$$

and

$$\zeta=\frac{c}{2m\omega} \quad (12)$$

which is called as damping ratio.

3.2. Shape Function of the Free Beam (Serbest Kirişin Şekil Fonksiyonu)

The four integration constants in the general solution of the characteristic shape function given in Eq. (7) are determined by the boundary conditions of the free beam shown in Fig. 1. The boundary conditions for such a beam are as follows:

$$at\ x=0 \quad \begin{cases} M(0,t)=0 & \text{or} \quad X''(0)=0 \\ V(0,t)=0 & \text{or} \quad X'''(0)=0 \end{cases}$$

$$at\ x=L \quad \begin{cases} M(L,t)=0 & \text{or} \quad X''(L)=0 \\ V(L,t)=0 & \text{or} \quad X'''(L)=0 \end{cases} \qquad (13)$$

Eqs. (13) gives a set of equations with constant coefficients. The determinant of the coefficients must be equal to zero for nontrivial solution. The expansion of this determinant leads to

$$\cosh\lambda L\cdot\cos\lambda L=1 \qquad (14)$$

which is the frequency equation for the free beam. The numerical solution of this transcendental equation gives with a good approximation the following relationship

$$\lambda L=(n+1/2)\pi \qquad n=1,2,3,... \quad (15)$$

For the first mode, that is *n = 1*, Eq. (15) yields a value of 4.71 for λL while the exact value is appr. 4.73. For upper modes the difference is getting smaller. In the computations, the exact solutions of Eq. (14) must be used for the first several modes (say 5 modes), while Eq. (15) may be utilized for higher modes. After finding the values of λL, the natural frequencies can be obtained from Eqs. (8) and (11) as follows:

$$\omega_n=\sqrt{\frac{\lambda_n^4 EI+k}{m}} \qquad (16)$$

The solution to the set of homogeneous equations (13) is parameter-dependent. However, normal modes are determined to a relative magnitude, therefore the constant arose in the solution may be taken unity. Hence, the characteristic shape function for the *n-th* mode is obtained as

$$X_n(x)=\sin\lambda_n x+\sinh\lambda_n x-\beta_n(\cos\lambda_n x+\cosh\lambda_n x) \qquad (17)$$

where

$$\beta_n=\frac{\sinh\lambda_n L-\sin\lambda_n L}{\cosh\lambda_n L-\cos\lambda_n L} \qquad (18)$$

From Eq. (3) the displacement function for the *n-th* mode is given by

$$y_n(x,t)=X_n(x)\cdot T_n(t) \quad (19)$$

The general solution to the equation of motion, namely the total deflection is obtained by superimposing all modes as follows:

$$y(x,t)=\sum_{n=1}^{\infty}X_n(x)\cdot T_n(t) \quad (20)$$

3.3. Forced Vibration (Zorlanmış Titreşim)

This paper deals with the transverse vibration of continuous beams on elastic soils subjected to dynamic disturbances due to concentrated loads and moments. However, the right hand side of Eq. (1) has been established for distributed loads. For this reason, concentrated loads will be transformed to distributed loads by the theory of generalized functions (distributions). The technique used in this study is to expand the Dirac distribution into a series of an orthogonal function family [11].

3.4. Foundation Beams under Concentrated Forces (Tekil Yük Etkisindeki Temel Kirişleri)

The concentrated force $F(s,t)$ on any position s of the beam may be transformed to distributed load by Dirac distribution as

$$f(x,t) = F_{0s}\phi_s(t)\delta_s(x) \tag{21}$$

where the load function is of the form

$$F(s,t) = F_{0s}\phi_s(t) \tag{22}$$

and

$$\delta_s(x) = \delta(x-s) = \sum_n A_n X_n(x) \tag{23}$$

In these equations F_{0s} is the amplitude of the force located at a point *s*, $\phi_s(t)$ is the time function of the force, $\delta_s(x)$ is Dirac distribution function centered at position *s*. This distribution is expanded into a series of shape functions. By taking the inner product of Eq. (23) with X_m, and utilizing the properties of distributivity, homogeneity and orthogonality, we obtain

$$A_n = \frac{\langle \delta_s, X_n \rangle}{\langle X_n, X_n \rangle} \tag{24}$$

The inner product of shape function of free beam for the same mode is

$$\langle X_n, X_n \rangle = \|X_n\|^2 = L\beta_n^2 \tag{25}$$

and from the definition of Dirac distribution [12]

$$\langle \delta_s, X_n \rangle = X_n(s) = X_{ns} \tag{26}$$

the constant A_n is obtained as

$$A_n = \frac{X_{ns}}{L\beta_n^2} \tag{27}$$

Substituting A_n into Eq. (23) yields

$$\delta_s(x) = \sum_n \frac{X_{ns}}{L\beta_n^2} X_n(x) \tag{28}$$

From Eqs. (21) and (28), the concentrated load $F(s,t)$ located at position *s* is transformed to distributed load as

$$f(x,t) = \frac{F_{0s}}{L}\phi_s(t)\sum_n \frac{1}{\beta_n^2} X_{ns} X_n(x) \tag{29}$$

It follows from this that the general differential equation for foundation beam under concentrated loads may be expressed as

$$EIy^{IV} + m\ddot{y} + c\dot{y} + ky = \frac{F_{0s}}{L}\phi_s(t)\sum_n \frac{1}{\beta_n^2} X_{ns} X_n(x) \tag{30}$$

By rearranging Eqs. (5), (8) and (16), and by the method of separation of variables, Eq. (30) reduces to

$$\ddot{T}_n + \frac{c}{m}\dot{T}_n + \omega_n^2 T_n = \frac{F_{0s}\phi_s(t) X_{ns}}{mL\beta_n^2} \tag{31}$$

The solution of the differential equation (31) is determined by Duhamel integral as follows:

$$T_n(t) = e^{-\zeta\omega t}[T_{n0}\cos(\omega_{Dn}t) + \frac{\dot{T}_{n0} + \zeta\omega T_{n0}}{\omega_{Dn}}\sin(\omega_{Dn}t)] + \frac{X_{ns}F_{0s}}{Lm\omega_{Dn}\beta_n^2}\int_0^t \phi_s(\tau)e^{-\zeta\omega(t-\tau)}\sin(\omega_{Dn}(t-\tau))d\tau \quad (32)$$

where T_{n0} and $\dot{T}_{n0}$ are parameters depending only on the initial conditions.

3.5. Foundation Beams under Concentrated Moment Loads (Tekil Moment Etkisindeki Temel Kirişleri)

Since Eq. (1) is arranged for distributed loads, the concentrated moment $M(s,t)$ on any position *s* of the beam, positive in clockwise direction, may be transformed to distributed load by Dirac distribution as

$$V(x,t) = \frac{\partial M(x,t)}{\partial x} = M_{0s}\psi_s(t)\delta_s(x) \quad (33)$$

and hence

$$f(x,t) = -\frac{\partial V(x,t)}{\partial x} = -M_{0s}\psi_s(t)\delta'_s(x) \quad (34)$$

where the moment function is of the form

$$M(s,t) = M_{0s}\psi_s(t)H_s(x) \quad (35)$$

In these equations M_{0s} is the amplitude of the moment located at a point *s*, $\psi_s(t)$ is the time function of the force, $\delta_s(x)$ is Dirac distribution function centered at position *s*, and $H_s(x)$ is Heaviside function. The first derivative of Dirac distribution may be expanded into a series of shape functions as

$$\delta'_s(x) = \delta'_s(x-s) = \sum_n D_n X_n(x) \quad (36)$$

By taking the inner product of Eq. (36) with X_m and taking into consideration the modal orthogonality together with the solution in Eq. (25), we obtain

$$D_n = \frac{\langle \delta'_s, X_n \rangle}{L\beta_n^2} \quad (37)$$

The inner product of *k-th* derivative of Dirac distribution with any function *g(x)* is given as

$$\int_0^L \delta^{(k)}(x-s)\cdot g(x) = \int_0^L \delta_s^{(k)}(x)\cdot g(x+s) = (-1)^k g^{(k)}(s) \quad (38)$$

from last two equations the constant D_n is found as

$$D_n = -\frac{X'_{ns}}{L\beta_n^2} \quad (39)$$

which leads to

$$\delta'_s(x) = -\sum_n \frac{X'_{ns}}{L\beta_n^2}X_n(x) \quad (40)$$

Consequently, from Eqs. (1), (34) and (40), the general differential equation for foundation beam on elastic soil under concentrated moment loads is obtained as

$$EIy^{IV} + m\ddot{y} + c\dot{y} + ky = \frac{M_{0s}}{L}\psi_s(t)\sum_n \frac{1}{\beta_n^2} X'_{ns} X_n(x) \tag{41}$$

which, by the method of separation of variables reduces to

$$\ddot{T}_n + \frac{c}{m}\dot{T}_n + \omega_n^2 T_n = \frac{M_{0s}\psi_s(t)X'_{ns}}{mL\beta_n^2} \tag{42}$$

The solution of the differential equation (42) is determined by Duhamel integral as follows:

$$\begin{aligned} T_n(t) &= e^{-\zeta\omega t}[T_{n0}\cos(\omega_{Dn}t) + \frac{\dot{T}_{n0} + \zeta\omega T_{n0}}{\omega_{Dn}}\sin(\omega_{Dn}t)] \\ &+ \frac{X'_{ns}M_{0s}}{Lm\omega_{Dn}\beta_n^2}\int_0^t \psi_s(\tau)e^{-\zeta\omega(t-\tau)}\sin(\omega_{Dn}(t-\tau))d\tau \end{aligned} \tag{43}$$

If more than one load acts on the system the generic equation to be solved may be written by superposition as

$$\ddot{T}_n + \frac{c}{m}\dot{T}_n + \omega_n^2 T_n = \frac{1}{mL\beta_n^2}\left[\sum_{s=1}^{i} F_{0s}X_{ns}\phi_s(t) + \sum_{s=1}^{j} M_{0s}X'_{ns}\psi_s(t)\right] \tag{44}$$

where *i* is the number of concentrated force and *j* is the number of concentrated moment acting on the beam.

3.6. Initial Conditions (Başlangıç Koşulları)

The values of displacement and velocity functions for the beam at $t = 0$ have to be transformed to the time function and the first derivative of the time function with respect to *x*, namely the initial conditions for the time function. Let the displacement and velocity functions at initial time be $u(x)$ and $v(x)$, respectively, that is

$$y(x,0) = u(x) \tag{45}$$

$$\dot{y}(x,0) = v(x) \tag{46}$$

Hence, from Eq. (20)

$$u(x) = \sum_n X_n(x)\cdot T_n(0) \tag{47}$$

and

$$v(x) = \sum_n X_n(x)\cdot \dot{T}_n(0) \tag{48}$$

By taking the inner product of last two equations with X_m in view of the modal orthogonality and Eq. (25), we obtain

$$T_n(0) = T_{n0} = \frac{\langle u, X_n\rangle}{L\beta_n^2} \tag{49}$$

and

$$\dot{T}_n(0) = \dot{T}_{n0} = \frac{\langle v, X_n\rangle}{L\beta_n^2} \tag{50}$$

The inner product of shape function given in Eq. (17) with an arbitrary constant yields zero, namely

$$\langle 1, X_n\rangle = \int_0^L X_n(x)dx = 0 \tag{51}$$

Therefore, the parameters T_{n0} and $\dot{T}_{n0}$ take the value of zero for constant displacement and velocity and shape functions do not represent the initial conditions. For this reason, in case of constant $u(x)$, this value has to be superimposed with the values $y(x,t)$ calculated from Eq. (20).

3.7. Internal Forces (Kesit Tesirleri)

After determining the displacements caused by external loads acting on the foundation beam, the slope $\theta(x,t)$, bending moment $M(x,t)$ and shear force $V(x,t)$ at any given position x and time t may be evaluated by the following well-known relationships and Eq. (20) :

$$\theta(x,t) = \frac{\partial}{\partial x} y(x,t) = \sum_n X'_n(x) \cdot T_n(t) \tag{52}$$

$$M(x,t) = -EI \frac{\partial^2}{\partial x^2} y(x,t) = -EI \sum_n X''_n(x) \cdot T_n(t) \tag{53}$$

$$V(x,t) = -EI \frac{\partial^3}{\partial x^3} y(x,t) = -EI \sum_n X'''_n(x) \cdot T_n(t) \tag{54}$$

As mentioned before, if the initial displacement function is constant, namely

$$u(x) = u_C = const. \tag{55}$$

because of the property given in Eq. (51), it follows that

$$y(x,t) = \sum_n X_n(x) \cdot T_n(t) + u_c \tag{56}$$

For constant displacement and velocity functions

$$\begin{aligned} &\theta(x,0) = 0 \\ &M(x,0) = 0 \\ &V(x,0) = 0 \end{aligned} \tag{57}$$

therefore, the values found in Eqs. (52), (53) and (54) would not change.

4. NUMERICAL APPLICATION (SAYISAL UYGULAMA)

As an application of the method, the foundation beam shown in Fig. 2 is considered. The beam is prismatic and has the following properties: the flexural rigidity EI = 3000 MNm^2, mass per unit length m = 2 kNs^2/m^2, base width b = 1.2 m. The subgrade modulus of the soil, K_0 on which the foundation rests is 50 MN/m^3. All the excitation frequencies, Ω are taken as 100 rad/s. The problem is solved without damping, i.e., the damping ratio ζ = 0. The solution of the problem under this data set is referred to as *base*.

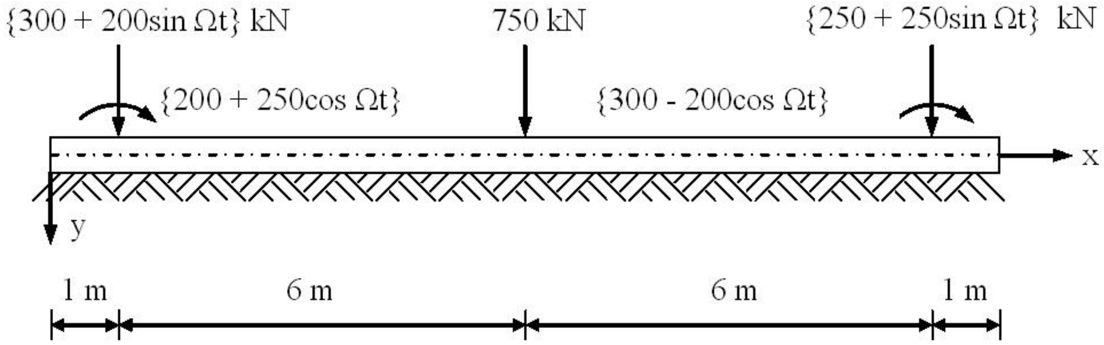

Figure 2. The foundation beam on elastic soil subjected to dynamic loading
(Şekil 2. Dinamik yüklemeye maruz elastik zemine oturan temal kirişi)

In order to compare the behavior of the foundation for different cases, the problem is solved for various data sets. In each set, one parameter is changed only. These parameters are the flexural rigidity, the subgrade modulus, the excitation frequencies and the damping ratio. Three

different values of each parameter used are shown in Table 1. Hata! Başvuru kaynağı bulunamadı.. Values of the parameters used in the different solutions
(Tablo 1. Farklı çözümlerde kullanılan parametrelerin değerleri)

Parameter	I	II	III
EI (MNm^2)	1000	10000	50000
K_0 (MN/m^3)	10	100	500
Ω (rad/s)	150	200	250
ζ (-)	5 %	10 %	20 %

The damping ratios in Table 1 may be defined from Eq. (12) as

$$\zeta = \frac{c}{2m\omega_1}$$

where ω_1 is the first natural frequency of the beam and is equal to 222.587 rad/s in the studied case.

The problem is solved by taking into account 200 modes for the interval of time corresponding to a duration of four periods which is approximately 0.1 s. The first natural period and the first five natural frequencies obtained for different values of parameters are set out in Table 2.

Table 3 gives the extremum values of displacements and bending moments of the foundation beam for different values of parameters. In this table, the positions and the time of occurrences of these extrama are shown too. Also relative changes (RC) which give comparisons between the results obtained from the base solution and the results obtained by changing the parameters are presented in the table.

Table 1. The first natural period (s) and the first five natural frequencies (rad/s) for different values of parameters
(Tablo 2. Farklı parametre değerleri için birinci doğal periyot(sn) ve ilk beş açısal frekans (rad/sn))

	T_1	ω_1	ω_2	ω_3	ω_4	ω_5
Base	0.02823	222.587	422.509	775.089	1260.815	1873.607
EI_1	0.03288	191.089	281.966	469.313	741.542	1090.933
EI_2	0.02037	308.465	724.600	1390.160	2286.668	3410.475
EI_3	0.01053	596.450	1582.792	3089.130	5101.397	7618.182
$K_{0,1}$	0.03931	159.828	393.082	759.449	1251.261	1867.191
$K_{0,2}$	0.02228	282.037	456.633	794.206	1272.657	1881.596
$K_{0,3}$	0.01112	565.283	669.712	933.147	1363.692	1944.326
D_1	0.02826	222.309	422.362	775.009	1260.766	1873.574
D_2	0.02837	221.472	421.922	774.770	1260.619	1873.474
D_3	0.02881	218.090	420.157	773.810	1260.029	1873.078

Table 2. The extremum values of displacements and bending moments of the foundation beam for different values of parameters

(Tablo 3. Farklı parametre değerleri için temel kirişinin deplasman ve eğilme momentleri ekstremum değerleri)

	y_{max}*			y_{min}†			M_{max}				M_{min}			
	Mag. (mm)	RC (%)	Time (10^{-2}s)	Mag (mm)	RC (%)	Time (10^{-2}s)	Mag. (kNm)	RC (%)	Pos (m)	Time (10^{-2}s)	Mag. (kNm)	RC (%)	Pos (m)	Time (10^{-2}s)
Base	2.57	-	8.32	-1.67	-	4.16	1022	-	6.86	4.47	-1014	-	10.04	2.17
EI_1	3.77	+47	2.96	-3.21	+92	5.37	866	-15	6.99	4.81	-805	-21	12.01	9.77
EI_2	1.07	-58	2.04	-1.10	-34	4.90	1433	+40	6.86	4.72	-1235	+24	10.04	2.11
EI_3	0.25	-90	2.14	-0.26	-84	5.73	1654	+62	7.10	4.74	-1296	+28	10.04	2.18
$K_{0,1}$	4.72	+84	7.71	-6.00	+259	5.27	2136	+109	7.00	5.24	-1396	+38	9.79	7.23
$K_{0,2}$	1.77	-31	2.03	-1.29	-23	5.17	927	-10	7.04	5.16	-905	-10	11.58	9.06
$K_{0,3}$	0.62	-76	2.26	-0.47	-72	6.02	649	-36	1.09	0.16	-686	-32	11.80	9.05
Ω_1	3.72	+45	5.31	-3.55	+113	3.78	1441	+41	6.97	3.55	-1191	+18	9.98	5.20
Ω_2	7.54	+193	7.70	-8.45	+406	9.20	2827	+177	7.09	9.20	-1795	+77	4.87	7.63
Ω_3	7.69	+199	9.48	-7.41	+343	8.17	2405	+135	7.04	8.20	-2102	+107	8.69	9.53
D_1	2.24	-13	2.29	-1.34	-20	4.19	893	-13	7.04	4.56	-906	-11	10.04	2.17
D_2	2.06	-20	2.30	-1.36	-19	5.17	810	-21	7.03	4.56	-821	-19	10.20	2.16
D_3	1.79	-30	2.33	-1.46	-13	5.47	722	-29	6.97	5.23	-706	-30	11.58	2.32

* maximum displacements occur at x = 14 m in each case; minimum displacements occur at x = 0 m
in each case

The deflection and bending moment responses of the foundation beam at x = L/2 obtained for different parameter values mentioned before are presented in Figs. 3 and 4, respectively.

Fig. 5 shows the elastic curves of the beam at the time in which the maximum positive deflection occurs for different parameter values. The instances given in Fig. 5 can also be seen in Table 3.

With the variation of parameters such as flexural rigidity, subgrade modulus, excitation frequency and damping ratio, it is clear from Table 3 and Fig. 5 that not only the magnitudes of the maximum deflections and bending moments but also their time of occurrences and their positions change. When Table 3 and Figs. 3 through 5 are perused, it can be observed that the influences of the variation of subgrade modulus and flexural rigidity on the dynamic responses are more pronounced compared to the variation of damping ratio and excitation frequency. The damping ratios used in this study are practical values. If higher damping ratios are considered, their influence on responses would be more distinguishable. As the excitation frequencies approach to the natural frequency of the system it is obvious that their responses increase sharply.

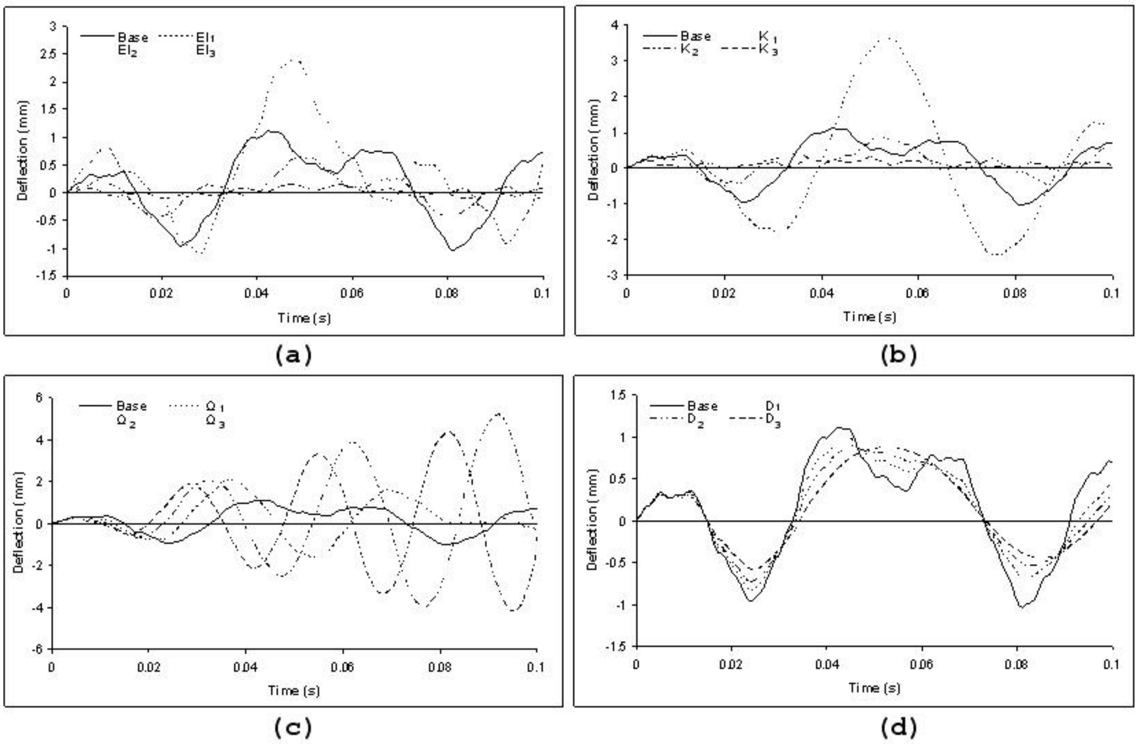

Figure 1. Deflection versus time of the beam shown in Fig. 1 at x = L/2 for different parameters: (a) flexural rigidity, (b) subgrade modulus, (c) excitation frequencies, (d) damping ratio
(Şekil 3. Şekil 1'deki kirişin (a) eğilme rijitliği, (b) yatak katsayısı, (c) zorlanmış titreşim frekansları, (d) sönüm oranı parametrelerine bağlı olarak x = L/2 kesitindeki deplasman - zaman ilişkisi)

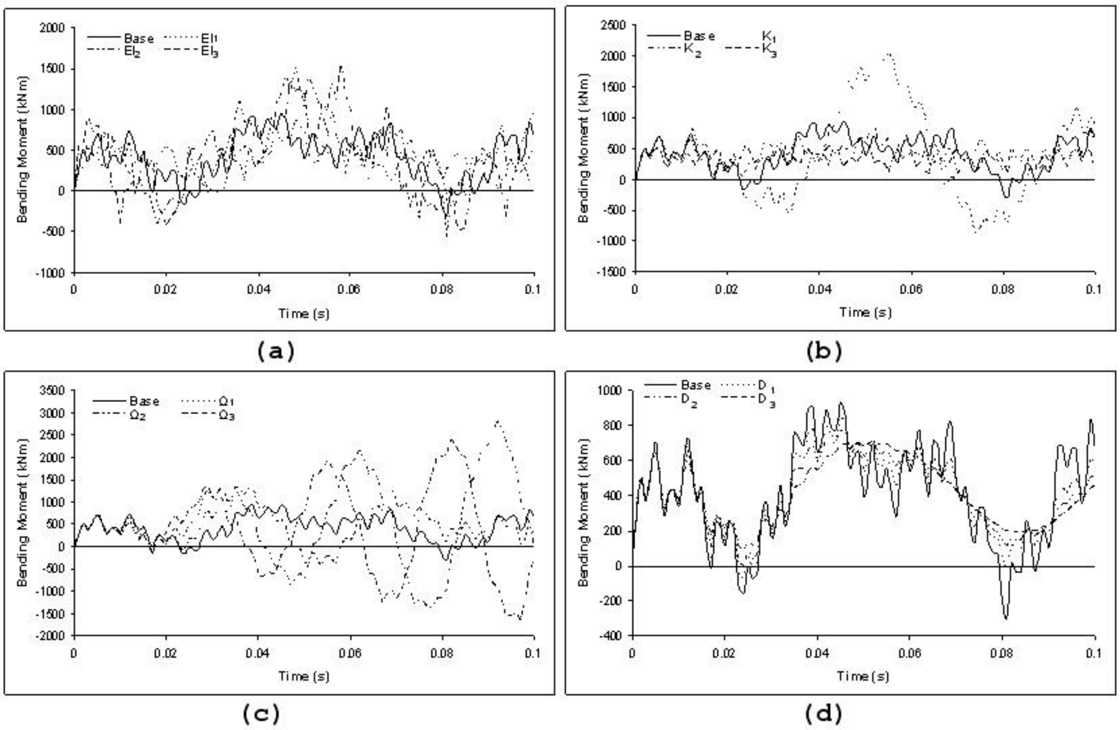

Figure 2. Bending moment versus time of the beam shown in Fig. 1 at x = L/2 for different parameters: (a) flexural rigidity, (b) subgrade modulus, (c) excitation frequencies, (d) damping ratio
(Şekil 4. Şekil 1'deki kirişin (a) eğilme rijitliği, (b) yatak katsayısı, (c) zorlanmış titreşim frekansları, (d) sönüm oranı parametrelerine bağlı olarak x = L/2 kesitindeki eğilme momenti - zaman ilişkisi)

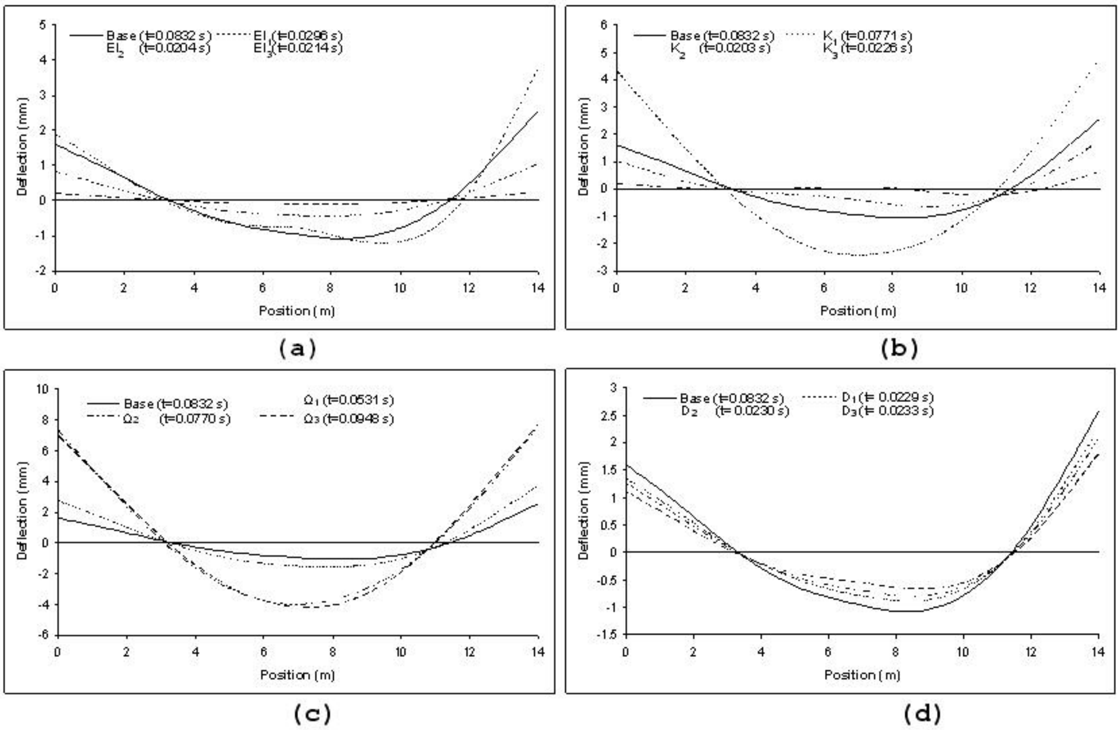

Figure 3. Elastic curve of the beam shown in Fig. 1 at the time in which the maximum positive deflection occurs for different parameters: (a) flexural rigidity, (b) subgrade modulus,(c) excitation frequencies, (d) damping ratio
(Şekil 5. Şekil 1'deki kirişin (a) eğilme rijitliği, (b) yatak katsayısı, (c) zorlanmış titreşim frekansları, (d) sönüm oranı parametrelerine bağlı olarak maksimum pozitif deplasmanın oluştuğu andaki elastik eğrisi)

5. CONCLUSIONS (SONUÇLAR)

The dynamic responses of a free beam subjected to transverse forces and moments on a Winkler foundation are presented. Since the governing differential equation is established for distributed loads, the concentrated forces and moments on beams have been transformed to distributed loads using Dirac distribution theory. Even though in theory this method is elegant, it turns out to be impractical in some cases. While the method yields reliable results for all dynamic responses (deflection, slope, bending moment and shear force) for concentrated forces, in the case of concentrated moment action, some inconsistencies may appear in shear forces due to the property of distribution functions. For the same reason, it is needed to involve a large number of modes to calculate shear forces for concentrated force and bending moments for concentrated moment loading. Since the distribution functions are expanded into a series of continuous shape functions, the discontinuities in the related internal forces at the points of application of the loads can be noticed only when higher modes are used.

REFERENCES (KAYNAKLAR)

1. Winkler, E., (1867). Die Lehre von der Elastizität und Festigkeit, Prag, 1867.
2. Keskinel, F. and Kumbasar, N., (1976). Continuous foundations and shells of revolution (in Turkish), Istanbul Technical University, Istanbul.
3. Filonenko-Borodich, M.M.,(1940). Some approximate theories of elastic foundation (in Russian). Uchenyie Zapiski Moskovskogo Gosudarstvennogo Universiteta, Mekhanika, U.S.S.R., 46 3-18.
4. Hetenyi, M., (1946). Beams on elastic foundation, The University of Michigan Press, Ann Arbor.
5. Kerr, A.D., (1964). Elastic and visco-elastic foundations models, Journal of Applied Mechanics, ASME, 31, pp: 491-498.
6. 6Pasternak, P.L., (1954). On a new method of analysis of an elastic foundation by means of two foundations constants (in Russian), Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu I Arkhitekture, Moscow, U.S.S.R.
7. Vlasov, V.Z. and Leontiev, N.N., (1960). Beams, plates and shells on elastic foundations(in Russian), Fizmatgiz Publishers, Moscow.
8. Daloglu, A.T. and Vallabhan, C.V.G., (2000). Values of k for slab on Winkler foundation, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 126, pp:463-473.
9. Misir, S., (2004). The effects of differential settlement on thestructural system (in Turkish), M. Sc. Thesis, Dokuz Eylül University, İzmir.
10. Terzaghi, K., (1966). Theoretical Soil Mechanics, 14th ed. John Wiley & Sons, Inc., New York.
11. Saatci, A., (1988). Dynamic analysis of building foundations and development of a computer program for practice (in Turkish), M. Sc. Thesis, Dokuz Eylül University, İzmir.
12. Arsac, J., (1966). Fourier transforms and the theory of distributions, Prentice-Hall, Inc., Englewood Cliffs, N.J.