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COMPLEX SOLUTIONS FOR FIFTH ORDER KdV EQUATION AND (3+1) DIMENSIONAL BURGERS EQUATION

ABSTRACT

In this paper, we implemented a direct algebraic method for the complex solutions of the fifth order Korteweg-de Vries (KdV) equation and (3+1) dimensional Burgers equation. By using this scheme, we found several complex solutions of the three fifth order KdV equations and (3+1) dimensional Burgers equation.
Keywords: Fifth Order Kdv Equation, (3+1) Dimensional Burgers Equation, Direct Algebraic Method, Complex Solutions, Traveling Wave Solutions

BEŞİNCİ MERTEBEDEN Kdv DENKLEMİ VE (3+1) BOYUTLU BURGERS DENKLEMİ İÇİN KOMPLEKS ÇÖZÜMLER

ÖZET

Bu çalışmada beşinci mertebeden KdV denklemi ve (3+1) boyutlu Burgers denkleminin kompleks çözümleri için Doğrudan Cebirsel Metodu sunacağız. Bu tekniği kullanarak beşinci mertebeden KdV denklemi ve (3+1) boyutlu Burgers denkleminin birkaç tane kompleks çözümünü bulacağız.
Anahtar Kelimeler: Beşinci Mertebeden Kdv Denklemi, (3+1) Boyutlu Burgers Denklemi, Doğrudan Cebirsel Metot, Kompleks Çözümler, Seyahat Eden Dalga Çözümleri

1. INTRODUCTION (GİRİŞ)

The theory of nonlinear dispersive wave motion has recently undergone much study. We do not attempt to characterize the general form of nonlinear dispersive wave equations [1 and 2]. Nonlinear phenomena play a crucial role in applied mathematics and physics. Furthermore, when an original nonlinear equation is directly calculated, the solution will preserve the actual physical characters of solutions [3]. Explicit solutions to the nonlinear equations are of fundamental importance. Various methods for obtaining explicit solutions to nonlinear evolution equations have been proposed. Many explicit exact methods have been introduced in literature [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23 and 24]. Among them are Generalized Miura Transformation, Darboux Transformation, Cole-Hopf Transformation, Hirota's dependent variable Transformation, the inverse scattering Transform and the Backlund Transformation, tanh method, sine-cosine method, Painleve method, homogeneous balance method, similarity reduction method, improved tanh method and so on. In fact, recently a direct algebraic approach has been constructed an automated tanh-function method by Parkes and Duffy [12]. The authors present a Mathematica package that deals with complicated algebraic and outputs directly the required solutions for particular nonlinear equations.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this study, we implemented a direct algebraic method [24] with symbolic computation to construct new complex solutions for fifth order KdV equation and (3+1) dimensional Burgers equation.

3. METHOD AND ITS APPLICATIONS (YÖNTEM VE UYGULAMALARI)

Before starting to give a direct algebraic method, we will give a simple description of the direct algebraic method [24]. For doing this, one can consider in a two variables general form of nonlinear PDE

$$Q(u,u_t,u_x,u_{xx},\ldots)=0\ , \tag{1}$$

and transform Eq.(2.1) with $u(x,t)=u(\xi)$,$\xi=ik(x-ct)$ and $\xi=ik(x-\alpha y-\beta z-\eta t)$,where k,c,α,β and η are real constants. After transformation, we get a nonlinear ODE for $u(\xi)$

$$Q'(u,-ikcu',iku',-k^2u'',\ldots)=0. \tag{2}$$

where $u'=\dfrac{du}{d\xi}$.

The solution of the equation (2) we are looking for is expressed in the form

$$U(\xi)=\sum_{m=0}^{n}a_mF^m(\xi), \tag{3}$$

where $\xi=ik(x-ct)$ and $\xi=ik(x-\alpha y-\beta z-\eta t)$, n is a positive integer that can be determined by balancing the highest order derivate and with the highest nonlinear terms in equation, a_m and ξ can be determined. Substituting solution (3) into Eq. (2) yields a set of algebraic equations for F^m and $(m=0,1,2,\ldots)$ then, all coefficients of F^m have to vanish. After this separated algebraic equation, we could found coefficients a_0,a_m and ξ. $F(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi)=b+F^2(\xi) \tag{4}$$

where $F' = \frac{dF}{d\xi}$ and b is a constant. Some of the solutions are given the paper [24].

In this work, we will consider to complex solution the fifth order KdV equation and (3+1) dimensional Burgers equation by using the direct algebraic method which is introduced by Huiqun Zhang [24].

EXAMPLE 1. (ÖRNEK 1)

Consider fifth order KdV equation,

$$u_t + u_x + uu_x + u_{xxx} + uu_{xxx} + u_{xxxxx} = 0, \tag{5}$$

For doing this example, we can use transformation with Eq. (1) then Eq. (5) become

$$-cu' + u' + uu' - k^2 u''' - k^2 uu''' + k^4 u^{(5)} = 0,$$

(6) When balancing uu''' with u^5 then gives *n=2*. Therefore, we may choose

$$u = a_0 + a_1 F + a_2 F^2 . \tag{7}$$

Substituting (7) into Eq. (6) yields a set of algebraic equations for a_0, a_1, a_2, b and c. These systems are finding as

$$\begin{aligned}
&a_1 b + a_0 a_1 b - a_1 bc - 2a_1 b^2 k^2 - 2a_0 a_1 b^2 k^2 + 16 a_1 b^3 k^4 = 0,\\
&a_1^2 b + 2a_2 b + 2a_0 a_2 b - 2a_2 bc - 2a_1^2 b^2 k^2 - 16 a_2 b^2 k^2 - 16 a_0 a_2 b^2 k^2 + 272 a_2 b^3 k^4 = 0, ,\\
&a_1 + a_0 a_1 + 3a_1 a_2 b - a_1 c - 8a_1 bk^2 - 8a_0 a_1 bk^2 - 18 a_1 a_2 b^2 k^2 + 136 a_1 b^2 k^4 = 0,\\
&a_1^2 + 2a_2 + 2a_0 a_2 + 2a_2^2 b - 2a_2 c - 8a_1^2 bk^2 - 40 a_2 bk^2 - 40 a_0 a_2 bk^2 - 16 a_2^2 b^2 k^2\\
&+1232 a_2 b^2 k^4 = 0,\\
&3a_1 a_2 - 6a_1 k^2 - 6a_0 a_1 k^2 - 48 a_1 a_2 bk^2 + 240 a_1 bk^4 = 0,\\
&2a_2^2 - 6a_1^2 k^2 - 24 a_2 k^2 - 24 a_0 a_2 k^2 - 40 a_2^2 bk^2 + 1680 a_2 bk^4 = 0,\\
&-30 a_1 a_2 k^2 + 120 a_1 k^4 = 0,\\
&-24 a_2^2 k^2 + 720 a_2 k^4 = 0.
\end{aligned} \tag{8}$$

From the solutions of the system, we can found

Case 1.

$$a_0 = \frac{1}{2}\left(3 + 40bk^2\right), a_1 = 0, a_2 = 30k^2, c = \frac{5 - 48b^2k^4}{2}, k \neq 0. \tag{9}$$

Case 2.

$$a_0 = \frac{29}{11},\ a_1 = 0,\ a_2 = 30k^2\ , b = \frac{5}{88k^2}\quad c = \frac{2345}{968},\ k \neq 0. \tag{10}$$

With the aid of Mathematica substituting (9) and (10) into (7), we have obtained the following exact complex traveling wave solutions of equation (5). These solutions are:

Case 1.

$$u_1 = \frac{1}{2}\left(30 + 40bk^2\right) + 30k^2\left(-\sqrt{-b}Tanh\left[\sqrt{-b}ik\left(x - \frac{5 - 48b^2k^4}{2}t\right)\right]\right)^2. \tag{11}$$

where b<0 and k is an arbitrary real constant.

$$u_2 = \frac{1}{2}\left(30 + 40bk^2\right) + 30k^2\left(-\sqrt{-b}Coth\left[\sqrt{-b}ik\left(x - \frac{5 - 48b^2k^4}{2}t\right)\right]\right)^2. \tag{12}$$

where b<0 and k is an arbitrary real constant.

$$u_3 = \frac{1}{2}\left(30 + 40bk^2\right) + 30k^2\left(\sqrt{b}Tan\left[\sqrt{b}ik\left(x - \frac{5 - 48b^2k^4}{2}t\right)\right]\right)^2. \tag{13}$$

where b>0 and k is an arbitrary real constant.

$$u_4 = \frac{1}{2}\left(30+40bk^2\right)+30k^2\left(-\sqrt{b}Cot\left[\sqrt{b}ik\left(x-\frac{5-48b^2k^4}{2}t\right)\right]\right)^2. \tag{14}$$

where b>0 and k is an arbitrary real constant.

$$u_5 = \frac{3}{2}+30k^2\left(-\frac{1}{ik\left(x-\frac{5}{2}t\right)}\right)^2. \tag{15}$$

where b=0 and k is an arbitrary real constant.

Case 2.

$$u_6 = \frac{29}{11}+30k^2\left(\sqrt{\frac{5}{88}}\frac{1}{k}Tan\left[\sqrt{\frac{5}{88}}i\left(x-\frac{2345}{968}t\right)\right]\right)^2. \tag{16}$$

where $b>0$, k is an arbitrary real constant.

$$u_7 = \frac{29}{11}+30k^2\left(-\sqrt{\frac{5}{88}}\frac{1}{k}\cot\left[\sqrt{\frac{5}{88}}i\left(x-\frac{2345}{968}t\right)\right]\right)^2. \tag{17}$$

where $b>0$, k is an arbitrary real constant.

EXAMPLE 2. (ÖRNEK 2)

Consider (3+1) dimensional Burgers equation,

$$\begin{aligned} &u_t-2uu_y-2vu_x-2wu_x-u_{xx}-u_{yy}-u_{zz}=0,\\ &u_x-v_y=0,\\ &u_z-w_y=0. \end{aligned} \tag{18}$$

For doing this example, we can use transformation with Eq. (1) then Eq. (18) become

$$\begin{aligned} &-i\eta u'+2i\alpha uu'-2ivu'-2iwu'+ku''+k\alpha^2u''+k\beta^2u''=0,\\ &u'+\alpha v'=0,\\ &-\beta u'+\alpha w'=0. \end{aligned} \tag{19}$$

When balancing uu', vu' and wu' with u'' then gives $n_1=1$, when balancing u' with v' then gives $n_2=1$ and when balancing u' with w' then gives $n_3=1$. Therefore, we may choose

$$\begin{aligned} &u=a_0+a_1F,\\ &v=b_0+b_1F,\\ &w=c_0+c_1F. \end{aligned} \tag{20}$$

Substituting (20) into Eq. (19) yields a set of algebraic equations for a_0,a_1,b_0,b_1,c_0,c_1. These systems are finding as

$$\begin{aligned} &-2ia_1bb_0-2ia_1bc_0+2ia_0a_1b\alpha-ia_1b\eta=0,\\ &-2ia_1bb_1-2ia_1bc_1+2a_1bk+2ia_1^2b\alpha+2a_1bk\alpha^2+2a_1bk\beta^2=0,\\ &-2ia_1b_0-2ia_1c_0+2ia_0a_1\alpha-ia_1\eta=0,\\ &-2ia_1b_1-2ia_1c_1+2a_1k+2ia_1^2\alpha+2a_1k\alpha^2+2a_1k\beta^2=0,\\ &a_1b+bb_1\alpha=0,\quad a_1+b_1\alpha=0,\quad bc_1\alpha-a_1b\beta=0,\quad c_1\alpha-a_1\beta=0. \end{aligned} \tag{21}$$

From the solutions of the system, we can found

$$a_1 = \frac{ik\alpha(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta},\ b_0 = \frac{1}{2}(-2c_0+2a_0\alpha-\eta),\ b_1 = -\frac{ik(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta},$$
$$c_1 = -\frac{ik\beta(1+\alpha^2+\beta^2)}{-1-\alpha^2+\beta}. \tag{22}$$

With the aid of Mathematica substituting (22) into (20), we have obtained the following exact complex traveling wave solutions of equation (18). These solutions are:

$$u_1 = a_0 + \frac{ik\alpha(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(-\sqrt{-b}Tanh\left[\sqrt{-b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$
$$v_1 = \frac{1}{2}(-2c_0+2a_0\alpha-\eta) - \frac{ik(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(-\sqrt{-b}Tanh\left[\sqrt{-b}ik(x-\alpha y-\beta z-\eta t)\right]\right) \tag{23}$$
$$w_1 = c_0 - \frac{ik\beta(1+\alpha^2+\beta^2)}{-1-\alpha^2+\beta}\left(-\sqrt{-b}Tanh\left[\sqrt{-b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$

where b<0 and k is an arbitrary real constant.

$$u_2 = a_0 + \frac{ik\alpha(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(-\sqrt{-b}Coth\left[\sqrt{-b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$
$$v_2 = \frac{1}{2}(-2c_0+2a_0\alpha-\eta) - \frac{ik(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(-\sqrt{-b}Coth\left[\sqrt{-b}ik(x-\alpha y-\beta z-\eta t)\right]\right) \tag{24}$$
$$w_2 = c_0 - \frac{ik\beta(1+\alpha^2+\beta^2)}{-1-\alpha^2+\beta}\left(-\sqrt{-b}\coth\left[\sqrt{-b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$

where b<0 and k is an arbitrary real constant.

$$u_3 = a_0 + \frac{ik\alpha(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(\sqrt{b}Tan\left[\sqrt{b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$
$$v_3 = \frac{1}{2}(-2c_0+2a_0\alpha-\eta) - \frac{ik(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(\sqrt{b}Tan\left[\sqrt{b}ik(x-\alpha y-\beta z-\eta t)\right]\right) \tag{25}$$
$$w_3 = c_0 - \frac{ik\beta(1+\alpha^2+\beta^2)}{-1-\alpha^2+\beta}\left(\sqrt{b}Tan\left[\sqrt{b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$

where b>0 and k is an arbitrary real constant.

$$u_4 = a_0 + \frac{ik\alpha(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(-\sqrt{b}Cot\left[\sqrt{b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$
$$v_4 = \frac{1}{2}(-2c_0+2a_0\alpha-\eta) - \frac{ik(1+\alpha^2+\beta^2)}{1+\alpha^2-\beta}\left(-\sqrt{b}Cot\left[\sqrt{b}ik(x-\alpha y-\beta z-\eta t)\right]\right) \tag{26}$$
$$w_4 = c_0 - \frac{ik\beta(1+\alpha^2+\beta^2)}{-1-\alpha^2+\beta}\left(-\sqrt{b}Cot\left[\sqrt{b}ik(x-\alpha y-\beta z-\eta t)\right]\right)$$

where b>0 and k is an arbitrary real constant.

4. CONCLUSION (SONUÇ)

In this paper, we implemented a direct algebraic method [24] with symbolic computation to construct new exact complex solutions for fifth order KdV equation and (3+1) dimensional Burgers equation. The method can be used to many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

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